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INDUCING LIQUIDITY IN THIN FINANCIAL MARKETS THROUGH COMBINED-VALUE TRADING MECHANISMS

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Abstract: Previous experimental research has shown that thin financial markets fail to fully equilibrate, in contrast with thick markets. A specific type of market risk is conjectured to be the reason, namely, the risk of partial execution of desired portfolio rearrangements in a system of parallel, unconnected double auction markets. This market risk causes liquidity to dry up before equilibrium is reached. To verify the conjecture, we organized markets directly as a portfolio trading mechanism, allowing agents to better coordinate their orders across securities. The mechanism is an implementation of the combined-value trading (CVT) system. We present evidence that our portfolio trading mechanism facilitates equilibration to the same extent as thick markets do. Like in thick markets, the emergence of equilibrium pricing cannot be attributed to chance. Inspection of order submission and trade activity reveals that subjects manage to exploit the direct linkages between markets presented by the CVT system.

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1 Introduction

This research explores the key result in [5] that thin financial markets (markets with only a few agents) fail to completely equilibrate. The result in [5] contrasts with that in [4], where thick financial markets were found to fully equilibrate. [5] conjectured that a certain type of market risk causes convergence problems in thin financial markets. To implement improved portfolio positions, agents have to coordinate their actions across several parallel, unconnected markets. The risk that orders would only partially be filled may have made agents hesitant to engage in further trading activity. Whence a reduction in liquidity as the end of trading approached and a halt in the convergence towards equilibrium. The conjecture links nicely with asset pricing theory, which posits that investors are merely interested in (payoffs on) portfolios of securities, rather than the component securities themselves. The value of an individual security is determined solely by its contribution to the risk and return of a portfolio. Beyond this, the risk and return of the security itself are irrelevant. That portfolio theory is relevant in agents' actual decision making is supported by evidence in [5, 6] that agents continuously attempt to gain expected return for a given volatility if such gains are possible.

The evidence in [5] did not come from field data. Instead, it was generated in experimental financial markets, which immediately raises the question whether the results are reliable. Payments in these experiments are relatively small in comparison with payoffs about which investors make decisions in the field and in comparison with lifetime wealth. It might be thought, therefore, that experimental financial markets do not provide sufficient incentive, and that observed failure to equilibrate is not surprising or significant. If incentives are too little, however, one wonders why thick experimental financial markets do fully equilibrate (see [4]). The only difference between them and the thin financial markets in [5] is numbers: only 6 to 15 sub-

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jects were in the thin markets experiments; up to 63 subjects participated in the thick markets experiments.[2] Further evidence on the adequacy of the incentives is provided in [6], where a series of certainty experiments are studied which expected utility theory would assert to be entirely equivalent to the experiments reported on here and in [4, 5]. In these (thick-market) certainty experiments, payoffs are determined according to the inferred quadratic utility functions that explain the pricing in the risky assets experiments. In effect, utility functions are assigned, so that one can directly measure distance from optimal behavior as utility losses. [6] finds that subject behavior is very far from optimal in early periods — in some experiments, foregone gains in early periods average as much as $350 on average — but behavior changes significantly over time, and is very close to optimal in later periods — on average only a couple of dollars is lost.[3] Apparently, the incentives are sufficient to force subjects to all but full optimum.

The contribution of this paper is to use experiments to explore the ability of a portfolio trading mechanism to overcome the alleged portfolio rebalancing problems in thin markets, thereby inducing sufficient liquidity for full equilibration.

We are not the first to suspect a deficiency in the standard trading system of parallel, unconnected markets. [15, 16] propose that index-contingent limit orders be introduced, as a first, efficient step towards full linkage of markets. In a standard game-theoretic model of market microstructure, these articles prove that the presence of index-contingent limit orders induces liquidity (measured as the price impact of an order of a particular size). We go all the way: agents can submit orders of arbitrary combinations of all individual securities, not just a pre-specified index. Unlike [15, 16], we determine empirically whether portfolio trading facilitates equilibration.

We measure the distance from equilibrium by means of a well-known asset pricing model, namely, the Capital Asset Pricing Model (CAPM). The CAPM makes a precise prediction about the relationship between prices of various securities for markets to be in equilibrium. The CAPM has been used successfully to measure equilibration in thick experimental financial markets. See [4]. The CAPM relies on mean-variance preferences. Because risk is fairly limited in an experimental setting, mean-variance approximations to subjects' actual preferences may be sufficiently reliable. Of course, this is ultimately an empirical issue.

[2] As a matter of fact, there was one other difference between the thin-market and thick-market experiments: the former were held in a multiple-unit computerized double auction system (a computerized version of the Chicago futures markets), whereas the latter used a computerized open book system (like the Paris Bourse). It is not clear whether the ability to see inframarginal limit orders in an open book system reduces the type of market risk that this paper focuses on. Ongoing research will clarify this point.

[3] Such large potential gains in early periods are possible because prices are typically *out of equilibrium*. Indeed, some subjects recognize this quite clearly and earn as much as $200 in experiments in which median earnings are only $50.

Under mean-variance preferences, the CAPM holds in equilibrium. Whence our focus on the CAPM. See also [10] for theoretical evidence that the CAPM obtains when risk is small. We complement graphical evidence of equilibration with formal tests. The tests give the probability that the actual evolution of our measures of distance from equilibrium could have emerged merely by chance. That is, we compute the probability that our results would have come about had prices been just changing randomly. Under a random walk, no economic forces (pressures of portfolio demand against a fixed supply) are at work. The only economic basis for a random walk is simple speculation, which would indeed make prices unpredictable.

Our trading mechanism, an implementation of Combined-Value Trading (CVT), is designed to cross heterogeneous portfolio orders. This is accomplished by a scale-back procedure that is reminiscent of the partial order filling in standard, one-security markets. The second ingredient of our trading mechanism is pricing. Markets need a clear, easily interpretable signal that reflects excess demand (price increases) or excess supply (price decreases). We use constrained, mixed linear-integer programming to determine prices and trades. The constraints are suggested by economic theory.

We confirm our conjecture that a portfolio trading mechanism induces full equilibration. Equilibrium generally emerges before volume decreases, indicating that any reduction in liquidity must be attributed to exhaustion of gains from trade. We record these results in an environment where [5] observed that markets did not fully equilibrate because liquidity dried up. Inspection of orders and trades reveals that subjects actively pursued the type of package trading that CVT allows for and that solves the market risk conjectured to be inherent in parallel, unconnected market systems.

The remainder of this paper is organized as follows. In the next section, we briefly describe how the results in [5] led to the conjecture that a portfolio trading mechanism may overcome arrests in the equilibration process. Section 3 discusses our measures of distance from equilibrium, which are used to determine the success of our portfolio trading mechanism. Section 4 describes the experimental setup. The portfolio trading mechanism itself is introduced in Section 5. Results and tests are reported in Section 6. Section 7 concludes.

2 Market Structure And Illiquidity In Thin Financial Markets

[5] studied a set of experiments designed to test equilibration in a repeated, multiple-asset market of two risky securities and one risk-free security. Cash was used as the medium of exchange. While the experiments revealed slow, steady convergence towards equilibrium, the equilibration often stopped short of the equilibrium. It was conjectured that equilibration halted because of subjects' hesitance to trade in the face

of market thinness.

The basis for this conjecture is the following. When an agent wishes to improve the risk/reward characteristics of her portfolio, she generally has to simultaneously trade in several markets at once. If some or all markets are thin, it is possible that only a strict subset of her orders will be executed. The resulting portfolio may therefore be different from the desired one, and even be inferior to the initial portfolio before the trade. Inferiority may follow if the risk-free security earns a positive interest rate, while the cash that one receives for security sales earns none. The risk of ending up with an inferior portfolio may cause an optimizing agent not to try to improve her position at all, refraining her from participating further, and hence, generating liquidity.

This can easily be illustrated graphically. For instance, consider the position in mean-volatility space indicated by the diamond in Figure 1. The corresponding portfolio consists of a combination of two risky securities (securities A and B), some cash, but no risk-free securities (security C). To be exact: 3 units of A, 3 units of B and 400 francs cash.[4] For comparison, the position of the three securities is indicated with circles. Consider now the portfolio indicated by the star. It sits on the efficient frontier (the solid line) and dominates the initial position (diamond): it has higher mean return, and lower variance. It is optimal for a certain level of risk aversion.[5] The starred portfolio consists of zero cash, 2.1 units of A, 1.5 units of B and 9.0 units of C. So, the investor may want to improve his/her portfolio's performance by selling A and B, and buying C. If, however, the investor succeeds in selling only holdings of security A, s/he moves down and left in mean-variance space, towards the position indicated by the square. While not immediately obvious, this entails a loss in utility.[6] This is because the sale of security A merely generates cash, which earns zero interest. Consequently, our investor may rationally decide not to engage in any attempt to improve his/her position, even if it is clearly dominated in mean-variance space. It is the transaction risk caused by market thinness that induces a seemingly satisficing attitude.

This means that the market mechanism may be at the heart of liquidity problems in small markets. The mechanism in [5] was one of parallel double auctions, where agents could not submit orders in one market contingent on events in another. To avoid moving from the diamond to the square in Figure 1, one would like to submit an order to buy 9 risk-free securities and sell 1.5 of B *conditional on* a sale of of 0.9 A. If such orders are possible, then even a risk averse agent would not refrain from participating in the trading.

[4] The prices of the 3 securities are: 200 (A), 190 (B), 95 (C); their expected payoffs are: 230 (A), 200 (B), 100 (C); the variances of A and B are 9867 and 1400, respectively, and their covariance is -1000.

[5] The utility function is linear in the mean and variance of the payoff; the coefficient of the mean is 1; that of the variance is $-0.5.10^{-4}$.

[6] The original utility level is 1648; the optimal level is 1673; the level after trading is 1642.

This calls for a portfolio trading mechanism: a system whereby agents can submit orders to trade packages of securities, instead of just one.

Most of the difficulties in implementing a test of our conjecture stem from the absence of flexible portfolio trading mechanisms.[7] The challenge was to design such a mechanism, which we will refer to as a combined-value trading system (CVT). This mechanism eliminates the natural tendency of parallel markets to decompose a portfolio and its valuation into its constituent parts. Rather, we allow traders to submit bids which reflect their desired multi-security portfolio transactions. Standard trading mechanisms involve orders by a player i which could be described with the pair (b_i, q_i). These are orders of the form "i is willing to pay up to b_i (respectively accept no less than b_i if the order is for a sale) for q_i units of a security." In contrast, CVT allows for orders which can be represented by the $N+1$-tuple $(b_i, \overrightarrow{q}_i)$, meaning "$i$ is willing to pay up to b_i for the vector of units of N securities $\overrightarrow{q}_i$." Additionally, agents can submit a scaling parameter, F_i. This scale indicates the minimal acceptable level at which a bid can be filled. A bid is now to be represented by the $N+2$-tuple $(b_i, \overrightarrow{q}_i, F_i)$, to be understood as "$i$ is willing to pay up to fb_i for the vector of units of N securities $f\overrightarrow{q}_i$, for any f between F_i and 1." We will discuss CVT in more detail in Section 5.

3 Measuring Equilibration

Given the limited size of the stakes in a typical experiment, we can safely assume that subjects' preferences towards risk can be approximated by mean-variance utility functions (provided of course that expected utility theory describes their attitudes towards risk in the first place). Hence, the Capital Asset Pricing Model (CAPM) would obtain in equilibrium. See also [10]. The CAPM predicts that equilibrium prices will be such that the market portfolio (i.e., the aggregate supply of risky securities) is optimal for mean-variance preferences. That is, the market portfolio generates maximum mean return for its volatility. See, e.g., [12, 11]. This prediction is independent of agents' levels of risk aversion, which is significant, because these cannot readily be measured, and moreover, may change during the course of the experiment.

Consequently, to determine whether experimental markets have equilibrated, we compare the reward-to-risk trade-off of the market portfolio against the maximum possible trade-off available at market prices. This trade-off is usually referred to as *Sharpe ratio*, to be defined as follows. Let R_{Ft} denote the return on a risk-free security at time t; let R_{mt} be the return on the market portfolio and let σ_{mt} denote its volatility.

[7] See [13, 14] for a mechanism that is related to ours. [13] reports experimental results from multi-security experiments, but mean-variance preferences were induced, effectively eliminating uncertainty. In our experiments, risk was explicit. Hence, subjects' natural inclinations in the face of uncertainty were the basis of trading and pricing.

The Sharpe ratio of the market portfolio is then

$$\frac{E[R_{mt} - R_{Ft}]}{\sigma_{mt}}$$

The maximal Sharpe ratio is the ratio of mean return in excess of the risk-free rate over volatility (when a riskfree security exists, the maximal Sharpe ratio is constant for all levels of volatility).[8]

At any moment in our experiments, we measure how far markets are from equilibrium by computing the difference between the market Sharpe ratio and the maximal Sharpe ratio. Markets reach equilibrium when the difference becomes zero. This measure was successfully employed in an experimental setting in [5, 4]. Notice that our distance measure can be computed without observation or estimation error. In field studies, lack of observability of the market portfolio makes it difficult to assess whether markets have equilibrated. Likewise, the payoff distribution and its parameters (means, variances, covariances) have to be estimated, and, hence, sampling error must be dealt with. In the laboratory, both the market portfolio and the payoff distribution are under control of the experimentor, and, therefore, known.[9]

The significance of our experimental results is further enhanced because (i) we made every possible effort to teach subjects beforehand about the nature of the payoff distribution, by means of pre-experiment learning sessions – see details below; (ii) we did not make endowments common knowledge, and avoided their becoming common knowledge by changing them across experiments (like in [5, 4]) as well as across periods within an experiment (unlike in [5, 4]). This means that subjects could not have deliberately used the CAPM to set prices and determine optimal portfolios[10]. Consequently, if we observe equilibration, it is only because of the economic forces that are at work, and not because of subjects' deliberate useage of the CAPM.

4 Experimental Design

We conducted a total of seven experiments, indexed in Table 1 by the date of the session. Subjects were recruited from the Caltech community, primarily undergraduates and a few graduate students from the

[8] In our computation of the maximal Sharpe ratio, we did take into account constraints on shortselling of risky securities. When shortsale constraints are binding, the maximal Sharpe ratio is not independent of volatility anymore. In that case, we use the maximal Sharpe ratio corresponding to the volatility of the market portfolio.

[9] There are additional problems with field studies, such as the necessity to assume that the payoff distribution can be estimated from observed payoff frequencies, and that agents knew this payoff distribution when setting prices, etc. None of these affect our interpretation of experimental data.

[10] Once in CAPM equilibrium, optimal portfolios can readily be constructed as a combination of the market portfolio and the risk-free security.

natural sciences and engineering. Because of the complexity of the trading interface and of the intricacies of securities trading, recruiting was limited to those who had taken or were in the midst of taking courses related to finance. Some subjects, particularly in later session, participated in more than one session. The number of subjects varied from a low of six to a high of fourteen.

We created three securities, denoted A, B, and C, each with a life of one period. At the end of the period, each security paid a single dividend and was then retired. The magnitude of the dividend depended on a random draw of one of three equally likely states, X, Y and Z. The state was drawn after the period was closed, so there was no insider, or asymmetric information. The payoff table was as follows.

	State		
Security	X	Y	Z
A	170	370	150
B	160	190	250
C	100	100	100

Notice that the dividend of A varies dramatically from state to state (with an expected value of 230), the dividend of B varies less and has an expected value of 200, and the dividend of C is constant at 100.

Each experiment consisted of multiple periods of similar trading conditions, varying only by the initial allocations given to subjects. At the beginning of a period, each player was supplied with some securities A and B, and some francs cash (an experimental currency whose conversion rate into dollars is indicated in Table 1). The period proceeded in a number of rounds, the first round three minutes in length and subsequent rounds ninety seconds long. During a round, subjects submit orders which are then displayed for all to see. At the end of each round, order submission was stopped while the allocation algorithm (described in Section 5) solved for all trades. These trades were executed, and net trade and market prices were announced. Then another round began. The number of rounds in a period varied between experiments. The November experiments had 10 rounds per period. As will be reported in Section 6, it appeared that markets equilibrated and subjects slowed their activity after 5 to 8 rounds. Therefore, the December experiments had only seven rounds per period. At the end of a period, one of the three states was chosen randomly and announced, players were paid their dividends according to the payoff table, and the period ended. Then, we began another period, with new allocations. In the case of the November experiments, the session consisted of six periods, and the December sessions were eight periods long. Subjects knew about the length of the session they were in.

Note that while the aggregate supply of securities A and B varied from period to period, security C was always in zero net supply. Additionally, while no short sales were permitted in A and B, players were allowed to short up to 8 units of C. Thus, the ability to buy A and B is not limited by the number of francs on hand. If a subject wishes to expand holdings beyond the bounds indicated by the endowment of francs, she could do so buy selling units of C and paying the dividend. A sale of C is equivalent to borrowing an amount equal to the sale price, in exchange for a repayment of 100 francs at the end of the period. To the buyer, the difference between the price paid and the dividend is a risk-free return since the payment is guaranteed. This sale and purchase of asset C determines the risk-free rate simultaneously with the rates of return on the risky securities. Since it is possible for players to lose money due to unfortunate draws of the state and to then declare bankruptcy, we needed to ensure the integrity of the incentive system. Any subject whose cumulative earnings were negative in two consecutive periods in an experimental session was asked to leave the experiment.

The initial allocations varied widely from experiment to experiment, as detailed in Table 1. While players were aware of their own initial allocations, they were not told the allocations of others. Therefore, the size of the market portfolio was unknown to any player. This is an important design consideration. As discussed in the previous section, it ensured that participants could not use the CAPM and Arrow-Debreu pricing model to deliberately set prices, thereby artificially generating the outcomes that we were looking for.

Subjects were informed that the experiments would last about three hours. Before assembling in the Caltech Social Science Experimental Laboratory, all subjects were given a URL with the instructions. At the end of each page of instructions is a short quiz, which had to be correctly completed before moving on to the next page. Once all pages were completed, subjects were given a URL for a practice experiment. Subjects were not allowed to participate in the three-hour in-lab experiment if they did not enter at least five practice bids in the practice experiment. Each subject who completed these tasks and arrived at the lab on-time received a \$10 bonus to their final earnings.

5 The Combined-Value Trading (CVT) Mechanism

We used a Combined-Value Trading (CVT) system in our experiments. In this environment, subject i submits an order of the form $(b_i, \overrightarrow{q}_i, F_i)$, read "$i$ is willing to pay up to fb_i for the vector of goods $f\overrightarrow{q}_i$, for any f between F_i and 1." With bids of this form, determining the payments (prices) and allocations that maximize gains from trade (surplus) and provide the incentives for traders to reveal their true willingness to pay is not as straightforward as in single-asset markets.

In our implementation, we allowed for a variety of order types. A multi-market order for agent i is a vector $< b_i, (q_i^A, q_i^B, q_i^C), F_i >$ where $b_i > 0$ means agent i is willing to pay at most b_i to buy the order and

$b_i < 0$ means agent i is willing to accept at least b_i for the order. Similarly, $q_i^j > 0$ means agent i wants to purchase up to q_i^j units of j in the order, and $q_i^j < 0$ means agent i wants to sell up to q_i^j units of j in the order. F_i is a scale factor ($0 \leq F_i \leq 1$) which indicates that agent i is willing to accept an order of the form $(f_i b_i, f_i q_i^A, f_i q_i^B, f_i q_i^C)$ where $f_i \in [F_i, 1]$. For an "all or none" bid, $F_i = 1$.

Orders for single securities can thereby easily be combined in one order as a package (portfolio). For example, a package order can specify a willingness to pay up to 1000 francs for 4 units of asset A, 2 units of asset B and 3 units of asset C, with full flexibility. This bid would be written $< 1000, (4, 2, 3), 0 >$. One form of packaged order is a *swap*, buying units of some assets and selling units of other assets. For example, a swap can specify a willingness to pay up to 100 francs to supply 3 units of asset A if and only if 1 unit of asset B is received, as well as a willingness to accept any scaled version of this bid. This bid would be $< -100, (3, -1, 0), 0 >$. Note that although the market system will not allow traders to bid more francs then they currently possess, a packaged bid with asset C can overcome this problem. Suppose a player has zero francs and is willing to pay 90 francs to supply of 1 unit of A for 2 units of B, $< 90, (1, -2, 0), 0 >$, which is a bid he cannot afford. In order to cover this transaction, he can offer to sell a unit of asset C, effectively borrowing the 90 francs for a promise of 100 francs at the end of the period. Now, his bid is $< 0, (1, -2, 1), 0 >$, which is within the player's budget constraint.

After the bids are called, we solve for the allocations and market prices. We will now use i to denote orders as opposed to agent identification to reduce the notational burdens. The *allocation* at each iteration is determined by solving the integer program:

$$\begin{array}{rll} & \max_{f_i} \sum_i b_i f_i & \\ \text{subject to} & \sum_i q_i^k f_i \leq 0, & k = \text{A,B,C} \\ & f_i \in [F_i, 1] \cup 0, & \forall i \end{array} \tag{1}$$

Note that the first constraint implies that the market will admit a net surplus. If there is a surplus of an asset in the solution, the market offers it for sale at the market price in the next round of the period.

While solving the allocation problem is fairly straightforward, calculating appropriate *market prices* is not as simple. After the allocation is computed in the above maximization problem, we know which bids will be matched and completed. But we also need to compute what each matched bid will pay or receive. That is, we need to compute the transaction prices. The principles we used in designing the pricing rule were that, (i) payments equal receipts among the bidders, (ii) no one pays more (receives less) than she bid (offered), (iii) there are incentives to reveal one's true willingness to pay, and (iv) everyone pays the same price per unit unless there are significant reasons for deviating. As we will see below, deviations from these principles will occur only in cases with important inflexibilities.

After the allocation problem is solved there are three categories of orders: (i) orders that were *accepted* by the allocation, (ii) orders that were *rejected* by the allocation, and (iii) orders that were *partially accepted* (that is, $0 < f_i^* < 1$, where f_i^* is the fraction of order i actually allocated). We treat the accepted part as an accepted order and the rejected part as a rejected order. Now take the entire collection of submitted orders as a quasi-linear economy and consider its competitive equilibrium. If a competitive equilibrium allocation and price vector p exist then the allocation would solve the maximization problem above and the prices p (one price for each item) would solve the following problem.

$$b_i - p \cdot q_i \geq 0 \quad \text{for all accepted orders}$$

$$b_i - p \cdot q_i \leq 0 \quad \text{for all rejected orders}$$

$$p \cdot \textstyle\sum_{i \in A} q_i = 0 \quad \text{(Walras' law)}$$

If such an equilibrium price exists, it would satisfy our principles and would be the natural price to set.

Unfortunately, neither the uniqueness nor the existence of such a price vector p is guaranteed. If the market equilibrium prices exist but are not unique, there are many ways to pick one. We use a reference pricing rule, which minimizes the difference between the equilibrium price and the reference price subject to satisfaction of our principles.

Non-existence is a deeper problem which can occur when subjects submit bids with flexibility levels other than zero, because it generates nonconvexities in a player's (revealed) preferences. As an example, consider Table 2, as well as the corresponding Figure 2 , which detail a bid schedule for a single asset. Suppose that bid 7 is to sell 3 units for at least 3 francs per unit and is an inflexible order ($F_7 = 1$). Further suppose bid 2 is to pay up to 4 francs per unit for 3 units and is fully flexible ($F_2 = 0$). Lastly, suppose bid 3 is to pay up to 2 francs per unit for 2 units, also flexibly. Surplus is maximized, given the flexibility constraints, if all bids except 4 and 8 are filled (3 is partially filled). There is, however, no competitive equilibrium. To see this, notice that at any price above 2 francs per unit bidder 3 is unwilling to buy units, but at any price below 3 per unit bidder 7 is unwilling to sell any units. There is no price so that demand equals supply, because the supply function effectively jumps where the demand function would cross.

To price the allocation when a competitive equilibrium does not exist, we construct a "pseudo-competitive equilibrium price." First, we ignore rejected orders and consider only the accepted orders, i.e., orders i such that $f_i^* > 0$. We then calculate a fully flexible allocation by maximizing the surplus, subject to no excess demand, with $f_i^* \in [0, 1]$. This is the allocation that would occur if all accepted orders were fully flexible. Next, we find prices for this allocation exactly as we did before; splitting the difference if the competitive equilibrium price is not unique. (In this case it is easy to show an equilibrium price will exist.) In the case of our example in Table 2, this means that the price is 3.50 francs: the price that would obtain if all orders were fully flexible ($F_i = 0, \forall i$).

But if we were to charge and pay every bid according to the price of 3.50, buyer 3 would be paying more than the maximum her bid indicated she was willing to pay per unit (2 francs). Further, even though seller 7 created the non-existence problem by requiring his bid be all-or-none, he would receive a surplus of 0.50 francs on the extra unit sold that way. To provide the right incentives, to minimize all-or-none bids where they are unnecessary, and to not over-charge or under-pay, we charge or pay each part of an originally accepted bid that was rejected in the fully flexible allocation exactly what they bid. So in Table 2, seller 7 will receive 3 francs for the last unit and buyer 3 will pay 2 francs for her unit.

But then the payments may not add up. If seller 7 receives 3 francs for the last unit sold and buyer 3 pays 2 francs for the unit bought we will have to pay out more than we receive. In fact, the area indicated as negative surplus in Figure 2 is exactly the amount we will be short. Let V^f denote the surplus from the fully flexible allocation, V^* denote the surplus from the original matching procedure, and let $dV = V^f - V^*$, the added surplus from flexibility. This dV is exactly the negative surplus in Figure 2. We need to collect this amount from those bids accepted in both the original and in the fully flexible allocation. There are many ways to carry out this accommodation of the inflexible bidders. We choose to charge the player that caused the accommodation to be necessary. In this case, $dV = 1$ franc. Charging this to seller 7 still leaves him better off than if he hadn't traded at all, and brings the market into balance. It is possible that even after charging the inflexible players for their accommodation that the market could still be out of balance. In this case, it becomes necessary to split prices. That is, we charge different prices to the buyers and sellers, with the aggregate difference designed to make up the market deficit.

It is important to note that there may be trading rounds in which no transactions take place. In these cases, the mechanism calculates prices at which no trade would be executed because it would generate negative profits. This is to ensure that even when no trade occurs, subjects receive information about the markets that enable trade to occur in the future.

The above may seem an extremely complex way to match and price combined value trades. It is made particularly difficult when one allows all-or-none trades. But the complexity is invisible to the agents, and is merely meant to provide them with clear signals: execution of trades as well as prices that reflect the economics of the situation. The success of the CVT mechanism is ultimately an empirical question. We measure its success in terms of the liquidity it generates in thin markets. We now turn to the results.

6 Results

We will discuss four aspects of the experimental outcomes: comparison of equilibration between CVT experiments and experiments with parallel markets, formal tests of equilibration, patterns in volume (liquidity), and allocations.

6.1 Equilibration

As described earlier, equilibration of financial markets will be measured using the predictions of the CAPM. In particular, we will examine the evolution of the difference between the maximal Sharpe ratio and that of the market portfolio. To measure the Sharpe ratio, we wait until at least one transaction has occurred in each market. Using the most recent transaction prices, we compute the maximum Sharpe ratio as well as the market's Sharpe ratio, and take the difference. This is then repeated after any call in which a transaction occurs. This produces a plot of the evolution of the difference between the maximum Sharpe ratio and that of the market. Figure 3 shows plots of the Sharpe ratio differences for our seven experiments.

With few exceptions, the (absolute) Sharpe ratio differences are less than 0.15. There is a tendency for the difference to be wider in earlier rounds of a period, but subsequently it narrows, which indicates that markets equilibrate. However, in later rounds of a period the difference sometimes widens, which seems to imply that the market moves off its equilibrium. We will document below that less trade occurs in later rounds of a period, making prices more sensitive to the few orders that are executed, or in the absence of trade, to the bid and ask prices. This causes the market to move away from CAPM equilibrium in later rounds. We will also report that reducing the number of rounds forces trading to occur earlier, implying that the prices in later rounds are no less arbitrary. This is confirmed in the last two frames of Figure 3, where the number of rounds is seven instead of ten (December experiments).

Subjects did have the tendency to submit orders that attempted to exploit mistakes that others may make. This was especially true for the risk-free asset (security C) during later rounds (when it hardly traded): while the theoretical no-arbitrage price is 100, some subjects invariably bid a low price for security C, and got it when others inadvertently submitted a low ask. Because such speculative bids somtimes impacted transaction prices, we decided to set the price of C equal to its no-arbitrage value (100). Sharpe ratios were computed on the basis of this theoretical price.

It should be emphasized that equilibration is far from a foregone conclusion in our markets. Subjects did not know the composition of the market portfolio, and, hence, could not use the CAPM to price securities, or to determine optimal investment strategies.

We set out to study how well our CVT mechanism does *relative to* the usual system of parallel, continuous double auctions with the same number of subjects. To gain perspective, Table 3 reports the averages of the distance between the Sharpe ratio of the market and the maximal Sharpe ratio, for each period in each experiment. Standard deviations are in parentheses. Within a period, the distance is clearly not independent over time: they display pronounced trending. See Figure 3. Therefore, we do not report usual standard errors (the standard deviation divided by the square root of the time series length), which would be a good measure of the estimation error of the sample mean for independent observations. We report the standard deviation instead. For comparison, Table 3 reports the median average distance for eight experiments with thin, parallel markets (from [5]) as well as six thick, parallel markets (from [4]).[11] Corresponding standard deviations are displayed in parentheses. Asterisks indicate the significance of the number of CVT experiments where the average distance is below the median of the parallel-market experiments.

In most cases (periods), the average distance between the Sharpe ratio of the market and the maximal Sharpe ratio in the CVT experiments is below the median of the thin-market experiments more often than expected by chance. The overall significance level is less than 1%, suggesting that the CVT mechanism outperformed the parallel, computerized double-auction system of the thin-market experiments. The significance of this finding is enhanced when one also takes into account that the CVT experiments were tougher in at least one respect: subjects' endowments were randomly changed across periods, whereas in the thin-market experiment, subjects were given the same initial allocation in all periods.[12] In contrast, the frequency that the average distance between the market Sharpe ratio and the maximum Sharpe ratio is below the median for the thick-market experiments is as expected by chance. That is, there is no statistical distinction between the CVT and thick-market experiments, despite the fact that the latter were ran with up to 63 subjects, whereas the former used at most 15 subjects.

6.2 Formal Tests Of Equilibration

We have relied on graphical evidence of equilibration, although the comparison with parallel-market experiments was made more formal. We now ask to what extent the evidence of equilibration in the CVT experiments was not a chance event. Even if they are not determined by any economics at all (beyond

[11] If the median fell between two outcomes, the lower of the two outcomes is reported instead of the median.

[12] Also, the numbers reported in Table 3 for the median average distance from equilibrium in the parallel-market experiments where below the 50th percentile if the latter did not coincide with a realization, while the statistical tests took them invariably to be the 50th percentile. This means that the tests are tougher. Finally, in one of the eight thin-market experiments, the aggregate risk was substantially less than in the others (and compared to the CVT experiments), so that Sharpe ratios were generally much lower.

absence of speculative opportunities), prices may accidentally be close to CAPM's prediction. The question is really: if prices at a point in time are far from CAPM's prediction, will they revert back? The test will effectively measure the chances that we read too much asset pricing theory in the data when in fact there is nothing going on besides speculation. Speculation will eliminate arbitrage opportunities, and, if subjects are risk neutral, will cause prices to behave like random walks. Even if prices are a random walk, with only three securities it is likely that the market portfolio accidentally becomes mean-variance efficient as predicted by the CAPM. We want to rule out that our observations are cases of mere luck.

Notice that our formal statistical test differs substantially from those used in the analysis of field data. Econometric techniques developed to test asset pricing theory on field data are simply not appropriate for the analysis of experimental data. In experiments, one controls crucial parameters that are unknown to the empiricist working with field data. Most importantly, the field empiricist does not know the parameters of the payoff distribution and the usual formal statistical tests build on this. Implementing such tests in an experimental setting would be schizophrenic, because the experimentalist does know the parameters of the payoff distribution.

The econometric analysis of financial markets experiments requires novel approaches. Perhaps the biggest challenge in developing new tests is the presence of nonstationarities in the data. For instance, within periods, the difference between the market's Sharpe ratio and the maximum Sharpe ratio (see Figure 3) exhibits trending, which makes it difficult to determine whether these differences are significant.

As a first step in the development of new tests, we propose to take the random walk hypothesis as the null, and test it against the hypothesis that the market is pulled towards the CAPM. Our test works as follows. Let $\Delta_{M,t}$ denote the distance between the Sharpe ratio of the market and the maximum Sharpe ratio, the subscript t denoting time (period and round). Consider the projection of the change in $\Delta_{M,t}$ onto $\Delta_{M,t-1}$:

$$\Delta_{M,t} - \Delta_{M,t-1} = \kappa \, \Delta_{M,t-1} + \epsilon_t. \tag{2}$$

where κ is such that ϵ_t is uncorrelated with $\Delta_{M,t-1}$. CAPM implies $\Delta_{M,t} = 0$; convergence to CAPM pricing implies $\kappa < 0$. We then determine the distribution of the least squares estimates of κ under the null hypothesis of a random walk, by randomly drawing from (bootstrapping) the empirical joint distribution of changes in transaction prices. The null hypothesis of a random walk is rejected in favor of stochastic convergence to CAPM if the least squares estimate of κ is beyond a critical value in the left tail of the ensuing distribution. This testing procedure is a variation of *indirect inference* (see [8]): we summarize the data in terms of a simple statistical model (in our case, a least squares projection) and determine the distribution of the estimates by simulating the variables entering the statistical model. Instead of simulating off a theoretical distribution, we bootstrap the empirical distribution, however.[13]

For each experiment, we estimated κ using OLS. We determined 5% and 10% critical values under the random walk null hypothesis by bootstrapping from the empirical joint distribution of price changes (we generated 200 price series of the same length as the sample used to estimate κ).[14]

Table 4 reports the results. The null of a random walk is rejected in one experiment at the 10% level, in four at the 5% level. There is less than a 1% chance to see four (or more) rejections at the 5% level out of seven. It is not surprising that we fail to reject the null in the two remaining experiments. Figure 3 suggests an explanation. The 11/4/99 Sharpe ratio distance behaves itself as a random walk. In the 11/30/99 experiment, the volatility is so low that given the opening prices one could easily have stuck on the frontier accidentally. In the five experiments in which we can reject the null of the random walk, the statistical analysis indicates that there is only a tiny probability to accidentally obtain the dynamics of Figure 3 if prices were indeed a random walk.

6.3 Volume

Figure 4 depicts the evolution of volume (in francs traded per subject) over time. In the ten-round experiments (11/99 experiments), volume remains even over the first six rounds, and declines subsequently. Cross-inspection with Figure 3 reveals that equilibration (if it occurs) generally completes before round six, and hence, before volume declines. This means that the reduction in liquidity (volume) in later rounds could reflect exhaustion of gains from trade. In the seven-round experiments (12/99 experiments), volume declines after two rounds, but does not dry up subsequently. Again, cross-inspection with Figure 3 reveals that equilibration is usually completed before volume declines.

We postulated that the CVT mechanism induces liquidity in thin experimental financial markets because subjects can rebalance portfolios more easily. To verify that subjects do indeed avail themselves of the added portfolio trading flexibility provided by CVT, Figures 5 and 6 report the percentage of orders submitted and executed that are combined-value (meaning that they involve at least two securities, as opposed to orders for one security against cash only). Both by round (Figure 5) and by period (Figure 6), between 20% and 30% of the orders are combined-value. The vast majority of these combined-value orders happen to be swaps (exchange of one or more securities for another). The success of our portfolio trading mechanism is further gauged in the finding that combined-value bids are no less likely to transact than are single-asset

[13] [7] also uses indirect inference, but, instead of matching an arbitrary statistical model, they match the scores of the likelihood function.

[14] We bootstrapped the mean-corrected empirical distribution, in order to stay with the null hypothesis of a random walk.

bids. Figures 5 and 6 report that the percentage of orders accepted that are combined-value is about equal to the percentage of orders submitted as combined-value.

6.4 Allocations

The CAPM also makes a precise allocational prediction: in equilibrium, investors all hold the same portfolio of risky securities, namely, the market portfolio. [4] reports that this does not even obtain in the thick-market experiments. Not only are subjects' end-of-period portfolios of risky securities different from the market portfolio, there is no convergence in later periods. This is particularly paradoxical, because the pricing result (CAPM pricing) is generally understood to critically depend on the allocational prediction: the market portfolio becomes mean-variance optimal only because every agent demands mean-variance optimal portfolios.[15]

We discovered the same price-allocation paradox in the CVT experiments. Figure 7 plots the proportion of security A in subjects' holdings of risky securities at the end of each period in all the experiments. The composition of the market portfolio is indicated with stars. Subjects' positions (dots) generally differ markedly from that of the market portfolio, and the differences do not diminish in later periods. So, thin-market CVT experiments and the thick-market experiments are alike in one other respect, because they both generate the price-allocation paradox.

7 Conclusion

In this paper, we report results from seven small-scale experimental financial markets where order submission and trading took place through a portfolio trading mechanism – the Combined-Value Trading (CVT) system. The results were compared to those from earlier experiments when markets were organized as a set of parallel double auctions. The new mechanism brings markets significantly closer to equilibrium ($p < 1\%$). The average distance from equilibrium is indistinguishable with that of large-scale experiments. Our findings cannot be attributed to chance: we reject that prices change randomly ($p < 1\%$), in favor of convergence towards the Capital Asset Pricing Model (CAPM).

Our results suggest that, to avoid illiquidity in thin (small-scale) financial markets, a portfolio trading mechanism should be used. The link between volume and portfolio trading is predicted by asset pricing theory, which posits that agents are not interested in securities individually, but in portfolios (packages

[15] See [6] for further analysis of the price-allocation paradox in thick markets and a theoretical model that explains why CAPM pricing can still obtain even if nobody buys a mean-variance efficient portfolio.

of securities). Unconnected, parallel double auctions do not allow agents to readily trade up to desired portfolio compositions, unless markets are sufficiently thick. The analysis of orders and transactions in the CVT experiments reveals that subjects did exploit the opportunity to trade portfolios rather than individual securities. Our results confirm the theoretical speculation in [15, 16].

The findings have implications for asset pricing theory, where illiquid assets are often thought of as generating an "equilibrium liquidity premium" over and above the usual risk premium. In view of the results of this paper, it is odd to think about an equilibrium liquidity premium, because illiquid markets appear to be associated with markets that do not equilibrate, and hence, an equilibrium liquidity premium cannot be envisaged.

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Table 1: List Of Experiments And Parameters

Date	Period	Number: Subjects	Number: Type	Endowment: A	Endowment: B	Endowment: C	Endowment: Cash (Francs)	Exchange Rate
11/1/99	1 - 5	6	3	9	1	0	400	0.02333
			3	1	9	0	400	0.02333
11/4/99	1 - 6	14	7	9	1	0	400	0.02333
			7	1	9	0	400	0.02333
11/11/99	1 - 4	11	4	9	1	0	400	0.02333
			7	1	9	0	400	0.02333
	5 - 6	10	4	9	1	0	400	0.02333
			6	1	9	0	400	0.02333
11/16/99	1	14	9	9	1	0	400	0.02333
			5	1	9	0	400	0.02333
	2	14	10	9	1	0	400	0.02333
			4	1	9	0	400	0.02333
	3	14	9	9	1	0	400	0.02333
			5	1	9	0	400	0.02333
	4	13	10	9	1	0	400	0.02333
			3	1	9	0	400	0.02333
	5	13	9	9	1	0	400	0.02333
			4	1	9	0	400	0.02333
	6	13	7	10	10	0	400	0.02333
			1	18	2	0	400	0.02333
			1	1	9	0	400	0.02333
			2	9	1	0	400	0.02333
			2	2	18	0	400	0.02333
11/30/99	1 - 4	15	10	9	1	0	400	0.02333
			5	1	9	0	400	0.02333
	5	14	10	9	1	0	400	0.02333
			4	1	9	0	400	0.02333
	6	14	9	9	1	0	400	0.02333
			5	1	9	0	400	0.02333

Date	Period	Number: Subjects	Number: Type	Endowment: A	Endowment: B	Endowment: C	Endowment: Cash (Francs)	Exchange Rate
12/2/99	1 - 5	12	6	6	3	0	400	0.03
			3	3	6	0	400	0.03
			3	4	5	0	400	0.03
	6	8	5	6	3	0	400	0.03
			3	4	5	0	400	0.03
	7	7	3	6	3	0	400	0.03
			3	3	6	0	400	0.03
			1	4	5	0	400	0.03
	8	7	3	6	3	0	400	0.03
			1	3	6	0	400	0.03
			3	4	5	0	400	0.03
12/7/99	1 - 2	14	9	9	1	0	400	0.03
			5	1	9	0	400	0.03
	3	13	10	9	1	0	400	0.03
			3	1	9	0	400	0.03
	4	13	8	9	1	0	400	0.03
			5	1	9	0	400	0.03
	5	13	8	9	1	0	400	0.03
			5	1	9	0	400	0.03
	6	13	10	9	1	0	400	0.03
			3	1	9	0	400	0.03
	7	13	8	9	1	0	400	0.03
			5	1	9	0	400	0.03
	8	6	5	9	1	0	400	0.03
			1	1	9	0	400	0.03

Table 2: Bid Submission List: Example

Bid Number (i)	Units (q_i)	Price (b_i)	F_i[a]
1	2	5.0	0
2	3	4.0	0
3	2	2.0	0
4	2	1.0	0
5	-1	-0.5	0
6	-2	-1.5	0
7	-3	-3.0	1
8	-3	-3.5	0

[a] Flexibility, equals 0 if full, and equals 1 if none.

Table 3: Average Distance From CAPM Equilibrium: CVT Experiments Against Thin And Thick Parallel-Market Experiments.

Experiment	Periods[a]							
	1	2	3	4	5	6	7	8
CVT markets:[b]								
11/01/99	0.11	0.10	0.14	0.14	0.11			
	(0.09)	(0.10)	(0.06)	(0.03)	(0.04)			
11/04/99	0.08	0.13	0.09	0.09	0.09	0.07		
	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.04)		
11/11/99	0.05	0.06	0.03	0.06	0.07	0.01		
	(0.01)	(0.08)	(0.05)	(0.03)	(0.10)	(0.02)		
11/16/99	0.27	0.09	0.04	0.07	0.07	0.09		
	(0.27)	(0.18)	(0.04)	(0.08)	(0.07)	(0.09)		
11/30/99	0.08	0.02	0.02	0.02	0.07	0.14		
	(0.05)	(0.02)	(0.01)	(0.04)	(0.06)	(0.10)		
12/02/99	0.32	0.02	0.01	0.03	0.01	0.12	0.05	0.15
	(0.28)	(0.03)	(0.02)	(0.02)	(0.01)	(0.11)	(0.03)	(0.13)
12/07/99	0.18	0.09	0.08	0.11	0.00	0.10	0.13	0.02
	(0.04)	(0.10)	(0.05)	(0.13)	(0.00)	(0.10)	(0.09)	(0.01)
thin markets[c]	0.17	0.10*	0.18**	0.17**	0.14**	0.15**	0.12	0.14
	(0.15)	(0.05)	(0.10)	(0.10)	(0.10)	(0.03)	(0.16)	(0.06)
thick markets[d]	0.20	0.07	0.06	0.07	0.04	0.08	0.06	0.03
	(0.18)	(0.07)	(0.06)	(0.07)	(0.02)	(0.30)	(0.09)	(0.02)

[a]Average difference between the maximum Sharpe (reward-to-risk) ratio and the Sharpe ratio of the market portfolio. Standard deviations in parentheses.

[b]Combined Value Trade (CVT) experiments, identified with date.

[c]Median (or one smaller if the median is not equal to a realized outcome) of the average difference between the maximum Sharpe (reward-to-risk) ratio and the Sharpe ratio of the market portfolio, experiments with thin parallel double auction markets (see [5]). Corresponding standard deviations in parentheses. An asterisk indicates that the number of average differences in the CVT experiments below the median in the thin-market experiments is significant at the 10% level. Two asterisks indicates significance at the 5% level. Overall significance: less than 1%.

[d]Median (or one smaller if the median is not equal to a realized outcome) of the average difference between the maximum Sharpe (reward-to-risk) ratio and the Sharpe ratio of the market portfolio, experiments with thick parallel open-book markets (see [4]). Corresponding standard deviations in parentheses. The number of average differences in the CVT experiments below the median in the thick-market experiments is nowhere significant at the 10% level.

Table 4: Test Of Random Walk Pricing Against CAPM Equilibrium

Experiment	Attraction Coefficient κ		
	Estimate[a]	Critical Value[b]	
		5%	10%
11/01/99	-0.0893^{*}	-0.1086	-0.0789
11/04/99	-0.0400	-0.1335	-0.1028
11/11/99	-0.3900^{**}	-0.1153	-0.0662
11/16/99	-0.3700^{**}	-0.0978	-0.0702
11/30/99	-0.0300	-0.0942	-0.0786
12/02/99	-0.2900^{**}	-0.1325	-0.0818
12/07/99	-0.2800^{**}	-0.1001	-0.0812

[a] Meaning of superscripts: ** = significant at the 5% level, * = significant at the 10% level. Overall significance: less than 1%.

[b] Based on 200 bootstrapped samples of the same size as used to estimate κ.

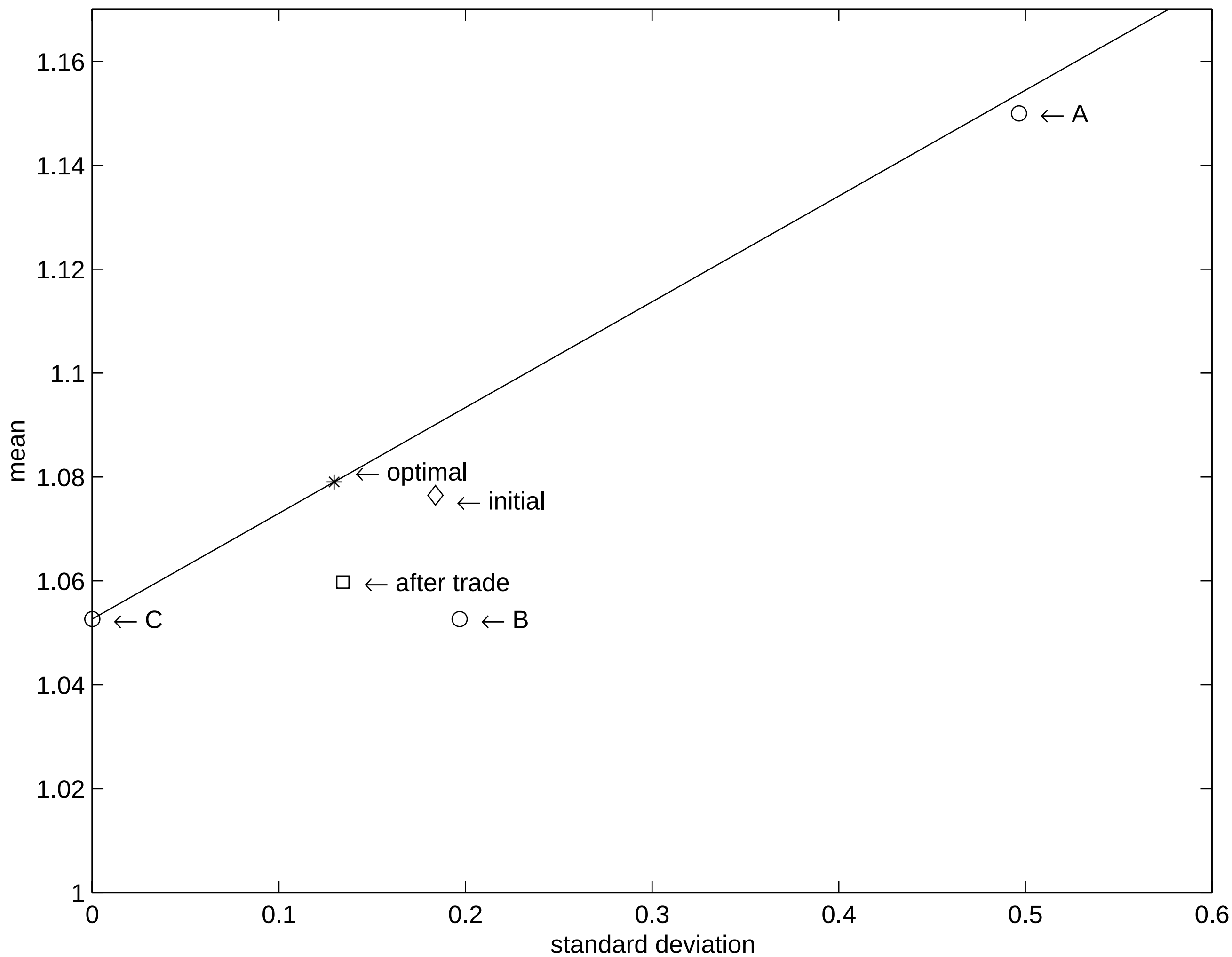

Figure 1: An investor who holds the position indicated by the diamond may want to move to the starred position, because it has a higher return and lower risk. If the investor manages to sell only A (receiving cash, which earns no interest), (s)he may move down to the position indicated with the square. If the starred position was indeed optimal, the new position is dominated by the original one.

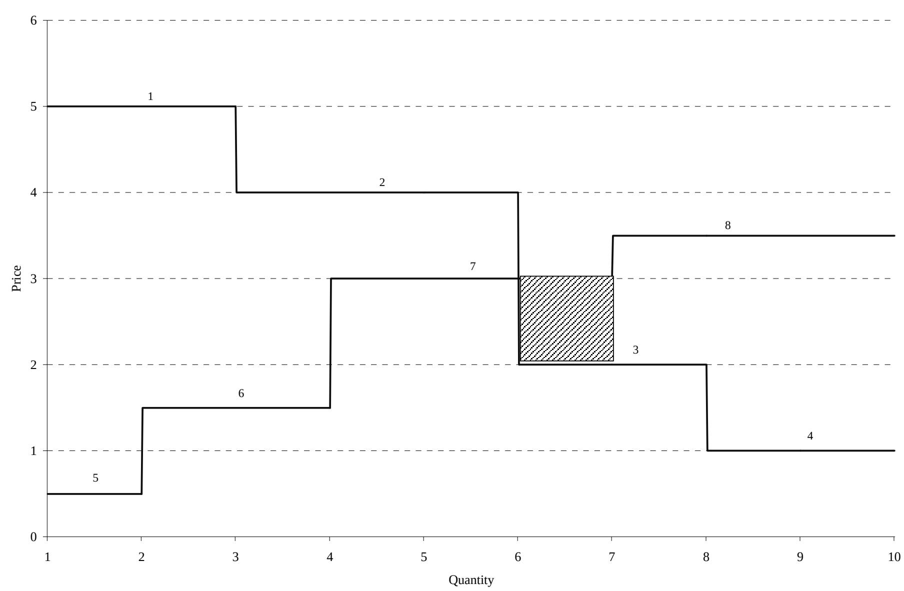

Figure 2: Supply-demand graph generated by the bids in Table 2. The shaded area represents the deficit caused by the inflexibility of bid 7.

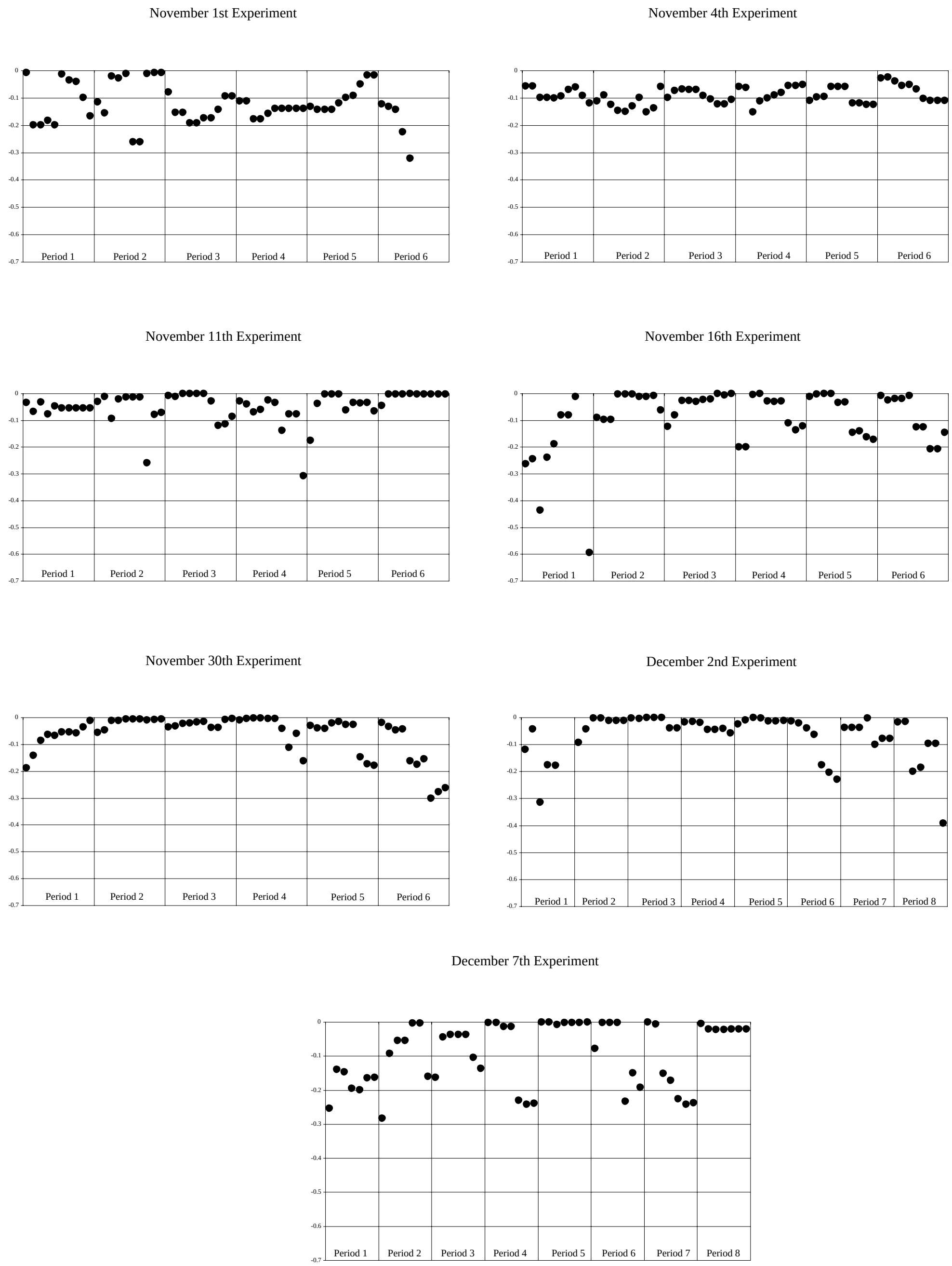

Figure 3: Evolution of the difference between the Sharpe ratio of the market portfolio and the maximum Sharpe ratio.

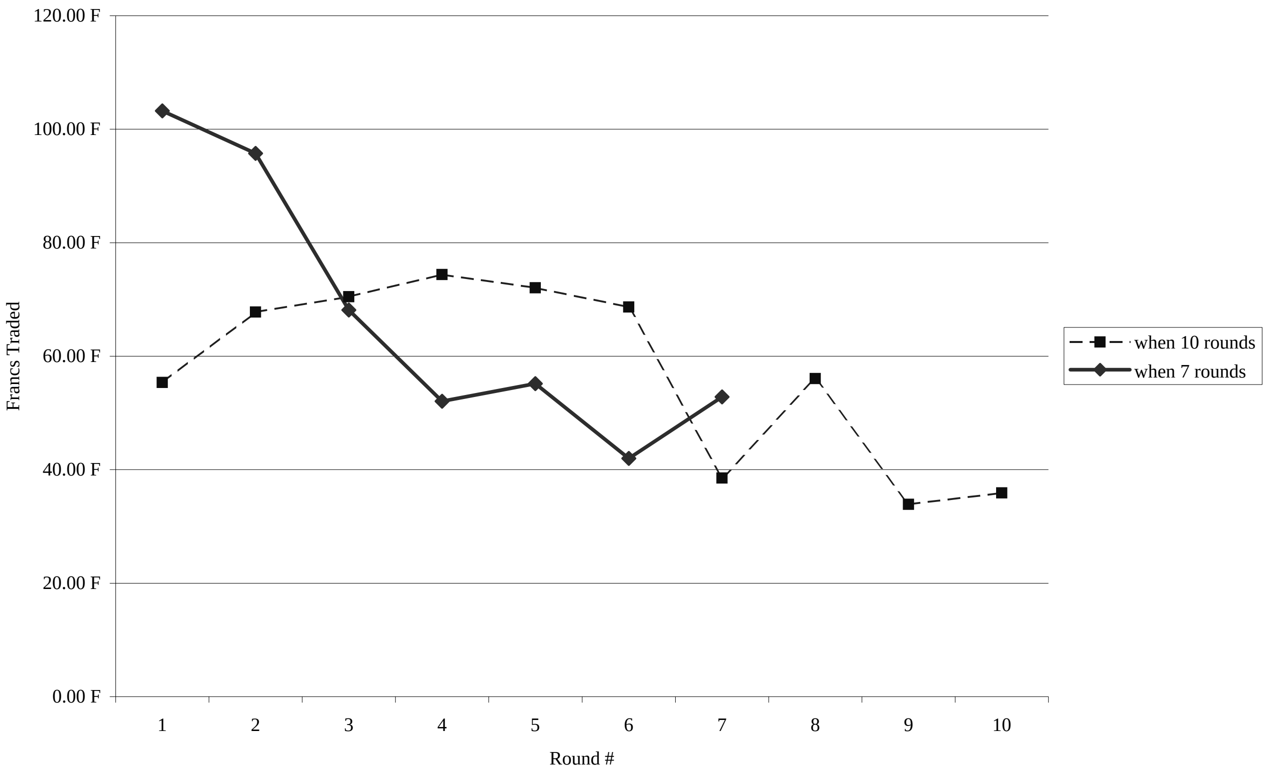

Figure 4: Per capita transaction volume (in francs), by round, averaged across periods and experiments.

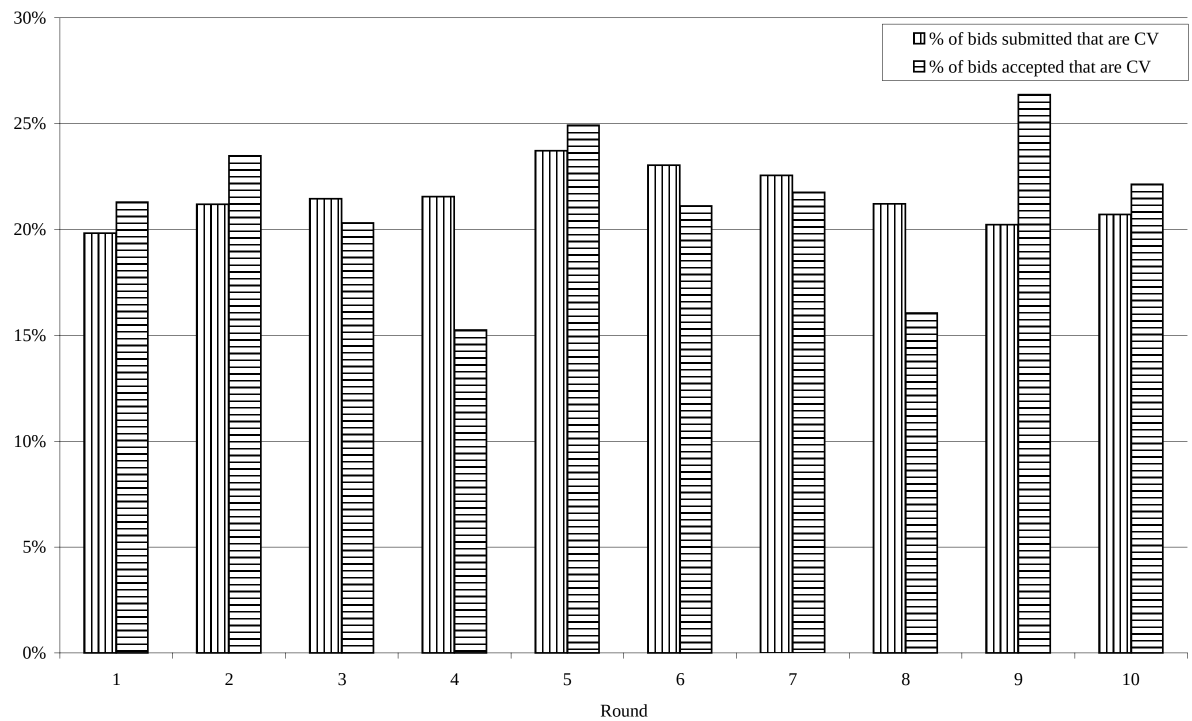

Figure 5: Percent of bids submitted and accepted that involve at least two securities, by *round*.

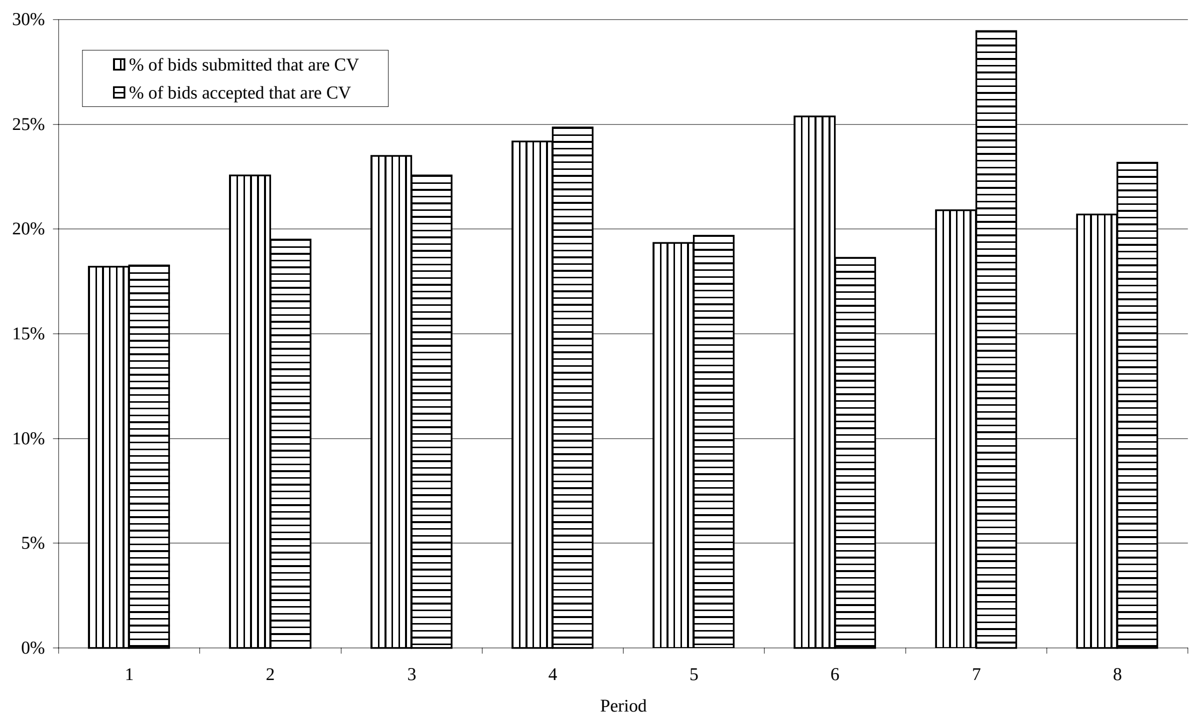

Figure 6: Percent of bids submitted and accepted that involve at least two securities, by *period*.

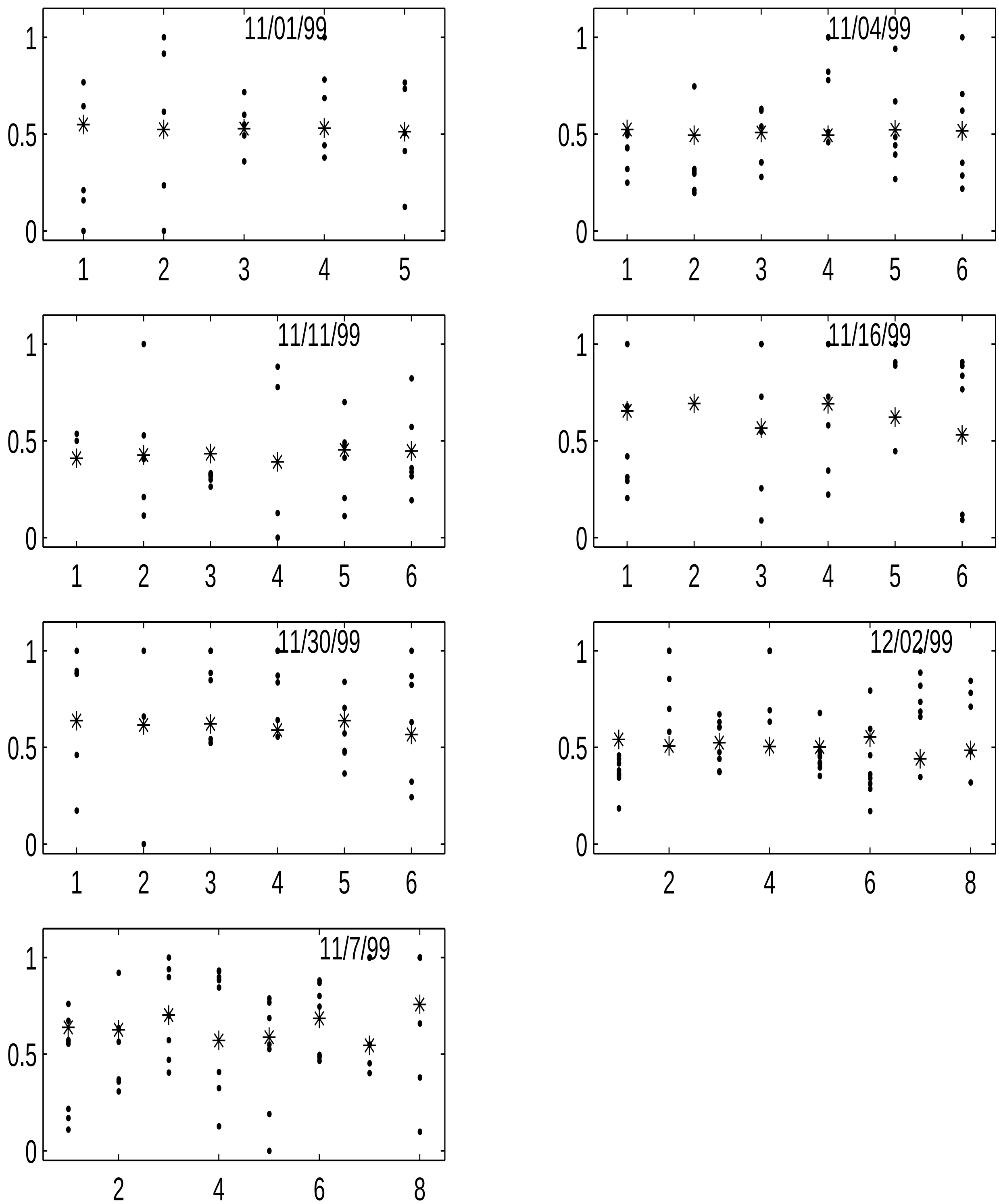

Figure 7: Fraction in subjects' portfolios of risky securities allocated to asset A (vertical axis), per period (horizontal axis), CVT experiments. Stars depict the composition of the market portfolio.